Reg. No. | | | | | | | | | |

BSCMTC 281

Choice Based Credit System IV Semester B.Sc. Degree
Examination, September 2022
(2020 – 21 Batch Onwards)
MATHEMATICS
Algebra and Complex Analysis

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **any ten** questions from Part **A**. **Each** question carries **2** marks.*
*2) Answers to Part **A** should be written in the **first few** pages of the answer book before answers to Part **B** and **C**.*
*3) Answer **six full** questions from Part **B** and **six full** questions from Part **C**.*
*4) Scientific calculators are **allowed**.*

PART – A

I. Answer **any ten** questions. **Each** question carries **2** marks. **(10×2=20)**

1) If G is a group and a, b, c $\in$ G, then prove the following :
   i) $ab = ac \Rightarrow b = c$ (Left Cancellation Law).
   ii) $ac = bc \Rightarrow a = b$ (Right Cancellation Law).

2) If G is a group with identity element e and $a^2 = e$, $\forall a \in G$, then prove that G is an abelian.

3) Prove that a group cannot be a union of its two proper subgroups.

4) Prove that every cyclic group is abelian.

5) Let $f : G \to G'$ be a homomorphism of groups. If G is abelian, then prove that G′ is also abelian.

6) Express the inverse of the cycle (1 2 4 5 3) as a product of transpositions.

7) Let $\mathbb{R}^*$ be the multiplicative group of non-zero reals. Define $f : \mathbb{R}^* \to \mathbb{R}^*$ by $f(x) = |x|$, $x \in \mathbb{R}^*$. Show that f is a group homomorphism and find its Kernel.

P.T.O.

8) Find the principal argument Arg(z) when $z = \frac{-2}{1+i\sqrt{3}}$.

9) Using de Moivre's formula, prove that $\cos(3\theta) = \cos^3(\theta) - 3\cos(\theta)\sin^2(\theta)$.

10) Show that $\lim_{z\to 0} \frac{z}{\bar{z}}$ does not exist.

11) Show that $f(z) = 3x + y + i(3y - x)$ is an entire function.

12) Solve $e^z = 1 + i\sqrt{3}$ for z.

13) Show that $\text{Log}(1 + i)^2 = 2\,\text{Log}(1 + i)$.

14) For $z = x + iy$, prove that $|\sin z| \geq |\sin x|$.

PART – B

II. Answer **any six** questions. **Each** question carries **5** marks. **(6×5=30)**

1) Let G be a group and H be a non-empty finite subset of G. Then prove that H is subgroup of G if and only if $ab \in H$, $\forall a, b \in H$.

2) Let H and K be subgroups of a group G. Then prove that HK is a subgroup of G, if and only if, HK = KH.

3) Define cyclic group and prove that an infinite cyclic group has exactly two generators.

4) Let G be a group and H be a non-empty finite subset of G. Then prove that any two left cosets of H in G are either disjoint or equal.

5) Prove that any non-cyclic group of order 4 is isomorphic to the klein 4-group.

6) Define group homomorphism. Let $f : G \to G'$ be a homomorphism of G onto G′. If G is cyclic, then prove that G′ is also cyclic.

7) Let $f : G \to G'$ be a homomorphism of groups. Then prove that Ker(f) is a subgroup of G. Also, prove that f is one-one if and only if Ker(f) = {e}.

8) State and prove the First Isomorphism Theorem.

9) Prove that every $\sigma \in S_n$ can be expressed as product of disjoint cycles.

PART – C

III. Answer **any six** questions. **Each** question carries **5** marks. **(6×5=30)**

10) Find the sixth roots of unity, exhibit them geometrically and identify the principal root.

11) Applying $\varepsilon - \delta$ definition of limit, show that $\lim_{z \to 0} \frac{(\bar{z})^2}{z} = 0$.

12) Show that the function $f(z) = |z|^2$ is continuous everywhere but differentiable only at the origin.

13) Suppose that $f(z) = u(x, y) + iv(x, y)$ is a function differentiable at the point $z_0 = x_0 + iy_0$. Then prove that the first order partial derivatives of u and v exist at the point (x_0, y_0) and they satisfies the equations $u_x = v_y$ and $u_y = -v_x$ at the point (x_0, y_0). Also $f'(z_0) = u_x(x_0, y_0) + iv_x(x_0, y_0)$.

14) Let $f(z) = u(x, y) + iv(x, y)$ be a function defined in some neighborhood of $z_0 = x_0 + iy_0$. If all the first order partial derivatives of u and v are continuous at (x_0, y_0) and they satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$ at the point (x_0, y_0). Then, prove that $f'(z_0)$ exists.

15) Find the harmonic conjugate of function $u(x, y) = \sinh x \sin y$ and the corresponding analytic function.

16) Show that the function $\overline{\exp(iz)} = \exp(i\bar{z})$ if and only if $z = n\pi$, $n \in \mathbb{Z}$.

17) For $z = x + iy$, prove the following :

    i) $|\sin z|^2 = \sin^2 x + \sinh^2 y$

    ii) $|\cos z|^2 = \cos^2 x + \sinh^2 y$.

18) Find the solutions of $\sinh z = i$.