Reg. No. | | | | | | | | | |

MT 505(b)

Third Semester M.Sc. Degree Examination, December 2018/January 2019
(Repeaters)
MATHEMATICS
Advanced Topology
Choice Based Credit System (Old Syllabus)

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **any five full** questions.*
*2) Answer to **each** full question shall **not** exceed **eight** pages of the answerbook. **No** additional sheets will be **provided** for answering.*
*3) **Use** of scientific calculator is **permitted**.*

1. a) Define a second countable space, a Lindelöf space and a separable space. Prove that a second countable space is Lindelöf and separable.

   b) Prove that the space $\mathbb{R}_l$ satisfies all the countability axioms excepting the second. **(7+7)**

2. a) Define a regular space. Prove that an arbitrary product of regular spaces is regular.

   b) Define a normal space. Prove that every regular Lindelöf space is normal. **(6+8)**

3. a) Show that a Hausdorff space need not be regular.

   b) Show that a subspace of a Normal space need not be normal. **(6+8)**

4. State and prove the Urysohn lemma. **14**

5. a) State and prove the Urysohn metrization theorem.

   b) Give an example to showing that a Hausdorff space with a countable basis need not be metrizable. **(9+5)**

6. a) Define a completely regular space. Prove that an arbitrary product of completely regular spaces is completely regular.

   b) Define a partition of unity dominated by a finite indexed open covering of a space X. Prove the existence of a partition of unity dominated by a finite indexed open covering $\{U_1, U_2, \ldots, U_n\}$ of a normal space X. **(7+7)**

P.T.O.

7. State and prove the Tychonoff theorem. **14**

8. a) Prove that every open covering $\mathcal{A}$ of a metrizable space X has a refinement that is an open covering of X and countably locally finite.

   b) Show that if a space X has a countable basis, then a collection $\mathcal{A}$ of subsets of X is countably locally finite if and only if it is countable. **(9+5)**

9. a) Let X be a regular space with basis $\mathcal{B}$ that is countably locally finite. Then prove that X is normal and that every closed set in X is a G_δ set in X.

   b) Define a paracompact space. Prove that every paracompact Hausdorff space is normal. **(8+6)**

10. a) Define the path homotopy relation between two paths in a space X and prove that it is an equivalence relation.

    b) Define a covering map. Let $p : E \to B$ be a covering map, let $p(e_0) = b_0$. Then prove that any path $f : [0, 1] \to B$ beginning at b_0 has a unique lifting to a path $\tilde{f}$ in E beginning at e_0.

    c) Show that any covering map is an open map. **(5+7+2)**
